Vector Algebra Formulae

If $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$ are orthonormal vectors and $\boldsymbol{A} = A_x\boldsymbol{i} + A_y\boldsymbol{j} + A_z\boldsymbol{k}$ then $|\boldsymbol{A}|^2 = A_x^2 + A_y^2 + A_z^2$. [Orthonormal vectors ≡ orthogonal unit vectors.]

Scalar product

$$\boldsymbol{A}\cdot\boldsymbol{B} = |\boldsymbol{A}|\,|\boldsymbol{B}|\cos\theta \qquad \text{where } \theta \text{ is the angle between the vectors}$$

$$= A_xB_x + A_yB_y + A_zB_z = [\,A_xA_yA_z\,]\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix}$$

Scalar multiplication is commutative: $\boldsymbol{A}\cdot\boldsymbol{B} = \boldsymbol{B}\cdot\boldsymbol{A}$.

Equation of a line

A point $\boldsymbol{r} \equiv (x, y, z)$ lies on a line passing through a point $\boldsymbol{a}$ and parallel to vector $\boldsymbol{b}$ if

$$\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b}$$

with λ a real number.

Equation of a plane

A point $\boldsymbol{r} \equiv (x, y, z)$ is on a plane if either

(a) $\boldsymbol{r}\cdot\widehat{\boldsymbol{d}} = |\boldsymbol{d}|$, where $\boldsymbol{d}$ is the normal from the origin to the plane, or

(b) $\dfrac{x}{X} + \dfrac{y}{Y} + \dfrac{z}{Z} = 1$ where X, Y, Z are the intercepts on the axes.

Vector product

$\boldsymbol{A}\times\boldsymbol{B} = \boldsymbol{n}\,|\boldsymbol{A}|\,|\boldsymbol{B}|\sin\theta$, where θ is the angle between the vectors and $\boldsymbol{n}$ is a unit vector normal to the plane containing $\boldsymbol{A}$ and $\boldsymbol{B}$ in the direction for which $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{n}$ form a right-handed set of axes.

$\boldsymbol{A}\times\boldsymbol{B}$ in determinant form

$$\begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$\boldsymbol{A}\times\boldsymbol{B}$ in matrix form

$$\begin{bmatrix} 0 & -A_z & A_y \\ A_z & 0 & -A_x \\ -A_y & A_x & 0 \end{bmatrix}\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix}$$

Vector multiplication is not commutative: $\boldsymbol{A}\times\boldsymbol{B} = -\boldsymbol{B}\times\boldsymbol{A}$.

Scalar triple product

$$\boldsymbol{A} \times \boldsymbol{B} \cdot \boldsymbol{C} = \boldsymbol{A} \cdot \boldsymbol{B} \times \boldsymbol{C} = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} = -\boldsymbol{A} \times \boldsymbol{C} \cdot \boldsymbol{B}, \quad \text{etc.}$$

Vector triple product

$$\boldsymbol{A} \times (\boldsymbol{B} \times \boldsymbol{C}) = (\boldsymbol{A} \cdot \boldsymbol{C})\boldsymbol{B} - (\boldsymbol{A} \cdot \boldsymbol{B})\boldsymbol{C}, \qquad (\boldsymbol{A} \times \boldsymbol{B}) \times \boldsymbol{C} = (\boldsymbol{A} \cdot \boldsymbol{C})\boldsymbol{B} - (\boldsymbol{B} \cdot \boldsymbol{C})\boldsymbol{A}$$

Non-orthogonal basis

$$\boldsymbol{A} = A_1\boldsymbol{e}_1 + A_2\boldsymbol{e}_2 + A_3\boldsymbol{e}_3$$

$$A_1 = \boldsymbol{\epsilon}' \cdot \boldsymbol{A} \quad \text{where} \quad \boldsymbol{\epsilon}' = \frac{\boldsymbol{e}_2 \times \boldsymbol{e}_3}{\boldsymbol{e}_1 \cdot (\boldsymbol{e}_2 \times \boldsymbol{e}_3)}$$

Similarly for A_2 and A_3.

Summation convention

$$\boldsymbol{a} = a_i\boldsymbol{e}_i \qquad \text{implies summation over } i = 1\ldots3$$

$$\boldsymbol{a} \cdot \boldsymbol{b} = a_i b_i$$

$$(\boldsymbol{a} \times \boldsymbol{b})_i = \varepsilon_{ijk} a_j b_k \qquad \text{where } \varepsilon_{123} = 1; \quad \varepsilon_{ijk} = -\varepsilon_{ikj}$$

$$\varepsilon_{ijk}\varepsilon_{klm} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$$