Trigonometric Formulae

$$\cos^2 A + \sin^2 A = 1 \qquad \sec^2 A - \tan^2 A = 1 \qquad \operatorname{cosec}^2 A - \cot^2 A = 1$$

$$\sin 2A = 2\sin A\cos A \qquad \cos 2A = \cos^2 A - \sin^2 A \qquad \tan 2A = \frac{2\tan A}{1-\tan^2 A}.$$

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B \qquad \cos A\cos B = \frac{\cos(A+B)+\cos(A-B)}{2}$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B \qquad \sin A\sin B = \frac{\cos(A-B)-\cos(A+B)}{2}$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B} \qquad \sin A\cos B = \frac{\sin(A+B)+\sin(A-B)}{2}$$

$$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2} \qquad \cos^2 A = \frac{1+\cos 2A}{2}$$

$$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2} \qquad \sin^2 A = \frac{1-\cos 2A}{2}$$

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2} \qquad \cos^3 A = \frac{3\cos A + \cos 3A}{4}$$

$$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2} \qquad \sin^3 A = \frac{3\sin A - \sin 3A}{4}$$

Relations between sides and angles of any plane triangle

In a plane triangle with angles A, B, and C and sides opposite a, b, and c respectively,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = \text{diameter of circumscribed circle.}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$a = b\cos C + c\cos B$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\frac{C}{2}$$

$$\text{area} = \frac{1}{2}ab\sin C = \frac{1}{2}bc\sin A = \frac{1}{2}ca\sin B = \sqrt{s(s-a)(s-b)(s-c)}, \qquad \text{where } s = \frac{1}{2}(a+b+c)$$

Relations between sides and angles of any spherical triangle

In a spherical triangle with angles A, B, and C and sides opposite a, b, and c respectively,

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

$$\cos a = \cos b\cos c + \sin b\sin c\cos A$$

$$\cos A = -\cos B\cos C + \sin B\sin C\cos a$$

Hyperbolic Functions Formulae

$$\cosh x = \frac{1}{2}(e^x + e^{-x}) = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots \qquad \text{valid for all } x$$

$$\sinh x = \frac{1}{2}(e^x - e^{-x}) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots \qquad \text{valid for all } x$$

$\cosh ix = \cos x$

$\sinh ix = i \sin x$

$\tanh x = \dfrac{\sinh x}{\cosh x}$

$\coth x = \dfrac{\cosh x}{\sinh x}$

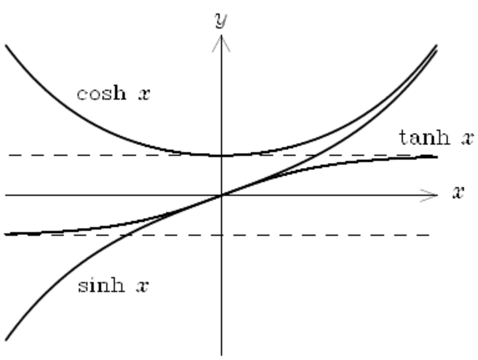
$\cosh^2 x - \sinh^2 x = 1$

$\cos ix = \cosh x$

$\sin ix = i \sinh x$

$\operatorname{sech} x = \dfrac{1}{\cosh x}$

$\operatorname{cosech} x = \dfrac{1}{\sinh x}$



For large positive x:

$\cosh x \approx \sinh x \to \dfrac{e^x}{2}$

$\tanh x \to 1$

For large negative x:

$\cosh x \approx -\sinh x \to \dfrac{e^{-x}}{2}$

$\tanh x \to -1$

Relations of the functions

$$\sinh x = -\sinh(-x) \qquad \operatorname{sech} x = \operatorname{sech}(-x)$$

$$\cosh x = \cosh(-x) \qquad \operatorname{cosech} x = -\operatorname{cosech}(-x)$$

$$\tanh x = -\tanh(-x) \qquad \coth x = -\coth(-x)$$

$$\sinh x = \frac{2\tanh(x/2)}{1-\tanh^2(x/2)} = \frac{\tanh x}{\sqrt{1-\tanh^2 x}} \qquad \cosh x = \frac{1+\tanh^2(x/2)}{1-\tanh^2(x/2)} = \frac{1}{\sqrt{1-\tanh^2 x}}$$

$$\tanh x = \sqrt{1-\operatorname{sech}^2 x} \qquad \operatorname{sech} x = \sqrt{1-\tanh^2 x}$$

$$\coth x = \sqrt{\operatorname{cosech}^2 x + 1} \qquad \operatorname{cosech} x = \sqrt{\coth^2 x - 1}$$

$$\sinh(x/2) = \sqrt{\frac{\cosh x - 1}{2}} \qquad \cosh(x/2) = \sqrt{\frac{\cosh x + 1}{2}}$$

$$\tanh(x/2) = \frac{\cosh x - 1}{\sinh x} = \frac{\sinh x}{\cosh x + 1}$$

$$\sinh(2x) = 2\sinh x \cosh x \qquad \tanh(2x) = \frac{2\tanh x}{1+\tanh^2 x}$$

$$\cosh(2x) = \cosh^2 x + \sinh^2 x = 2\cosh^2 x - 1 = 1 + 2\sinh^2 x$$

$$\sinh(3x) = 3\sinh x + 4\sinh^3 x \qquad \cosh 3x = 4\cosh^3 x - 3\cosh x$$

$$\tanh(3x) = \frac{3\tanh x + \tanh^3 x}{1+3\tanh^2 x}$$

$$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$$

$$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$$

$$\tanh(x \pm y) = \frac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$$

$$\sinh x + \sinh y = 2\sinh\frac{1}{2}(x+y)\cosh\frac{1}{2}(x-y) \qquad \cosh x + \cosh y = 2\cosh\frac{1}{2}(x+y)\cosh\frac{1}{2}(x-y)$$

$$\sinh x - \sinh y = 2\cosh\frac{1}{2}(x+y)\sinh\frac{1}{2}(x-y) \qquad \cosh x - \cosh y = 2\sinh\frac{1}{2}(x+y)\sinh\frac{1}{2}(x-y)$$

$$\sinh x \pm \cosh x = \frac{1 \pm \tanh(x/2)}{1 \mp \tanh(x/2)} = e^{\pm x}$$

$$\tanh x \pm \tanh y = \frac{\sinh(x \pm y)}{\cosh x \cosh y}$$

$$\coth x \pm \coth y = \pm\frac{\sinh(x \pm y)}{\sinh x \sinh y}$$

Inverse functions

$$\sinh^{-1}\frac{x}{a} = \ln\left(\frac{x+\sqrt{x^2+a^2}}{a}\right) \qquad \text{for } -\infty < x < \infty$$

$$\cosh^{-1}\frac{x}{a} = \ln\left(\frac{x+\sqrt{x^2-a^2}}{a}\right) \qquad \text{for } x \geq a$$

$$\tanh^{-1}\frac{x}{a} = \frac{1}{2}\ln\left(\frac{a+x}{a-x}\right) \qquad \text{for } x^2 < a^2$$

$$\coth^{-1}\frac{x}{a} = \frac{1}{2}\ln\left(\frac{x+a}{x-a}\right) \qquad \text{for } x^2 > a^2$$

$$\operatorname{sech}^{-1}\frac{x}{a} = \ln\left(\frac{a}{x}+\sqrt{\frac{a^2}{x^2}-1}\right) \qquad \text{for } 0 < x \leq a$$

$$\operatorname{cosech}^{-1}\frac{x}{a} = \ln\left(\frac{a}{x}+\sqrt{\frac{a^2}{x^2}+1}\right) \qquad \text{for } x \neq 0$$