Treatment of Random Errors

Sample mean $\bar{x} = \frac{1}{n}(x_1 + x_2 + \cdots x_n)$

Residual: $d = x - \bar{x}$

Standard deviation of sample: $s = \frac{1}{\sqrt{n}}(d_1^2 + d_2^2 + \cdots d_n^2)^{1/2}$

Standard deviation of distribution: $\sigma \approx \frac{1}{\sqrt{n-1}}(d_1^2 + d_2^2 + \cdots d_n^2)^{1/2}$

Standard deviation of mean:

$$\sigma_m = \frac{\sigma}{\sqrt{n}} = \frac{1}{\sqrt{n(n-1)}}(d_1^2 + d_2^2 + \cdots d_n^2)^{1/2}$$

$$= \frac{1}{\sqrt{n(n-1)}}\left[\sum x_i^2 - \frac{1}{n}\left(\sum x_i\right)^2\right]^{1/2}$$

Result of n measurements is quoted as $\bar{x} \pm \sigma_m$.

Range method

A quick but crude method of estimating σ is to find the range r of a set of n readings, i.e., the difference between the largest and smallest values, then

$$\sigma \approx \frac{r}{\sqrt{n}}.$$

This is usually adequate for n less than about 12.

Combination of errors

If $Z = Z(A, B, \ldots)$ (with A, B, etc. independent) then

$$(\sigma_Z)^2 = \left(\frac{\partial Z}{\partial A}\sigma_A\right)^2 + \left(\frac{\partial Z}{\partial B}\sigma_B\right)^2 + \cdots$$

So if

(i) $Z = A \pm B \pm C$, $\quad (\sigma_Z)^2 = (\sigma_A)^2 + (\sigma_B)^2 + (\sigma_C)^2$

(ii) $Z = AB$ or A/B, $\quad \left(\frac{\sigma_Z}{Z}\right)^2 = \left(\frac{\sigma_A}{A}\right)^2 + \left(\frac{\sigma_B}{B}\right)^2$

(iii) $Z = A^m$, $\quad \frac{\sigma_Z}{Z} = m\frac{\sigma_A}{A}$

(iv) $Z = \ln A$, $\quad \sigma_Z = \frac{\sigma_A}{A}$

(v) $Z = \exp A$, $\quad \frac{\sigma_Z}{Z} = \sigma_A$