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Mechanics Homework

Question 1:

Linear momentum and impulse

A child bounces a super ball on a side walk. The linear impulse delivered by the side walk to the super ball is 2 N.s during 1/800 sec of contact. What is the magnitude of the average force exerted on the super ball?

Solution

Given: $I = 2\text{Ns}$ and $\Delta t = \dfrac{1}{800}\text{sec}.$

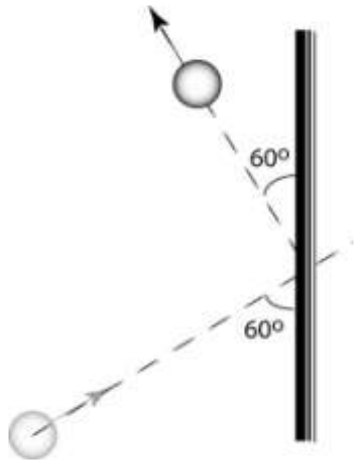
By definiton:

$I = F_{av}\Delta t$

Therefore the average force exerted on the superball is, $F_{av} = \dfrac{2}{1/800} = 1600\text{N}.$

Question 2:

A 3kg steel ball strikes a massive wall with a speed of 10m/s at an angle of 60^0 with the angle as shown. If the ball is in contact with the wall for 0.2 sec what is the average force exerted on the ball by the wall.

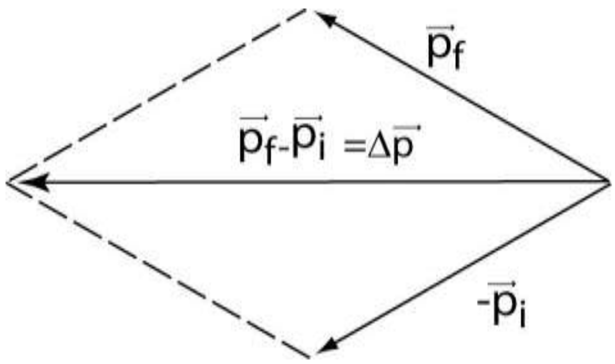


Solution:

$$\frac{\Delta p}{\Delta t} = F_{av} = \frac{\sqrt{p_f{}^2 + p_i{}^2 + p_i p_f}}{\Delta t}$$

$$= \frac{\sqrt{30^2 + 30^2} + 2\times 30\times 30\cos}{0.02}$$

$$= 260N. \text{ in the horizontal direction.}$$



Question 3:
A grenade flying horizontally with a velocity of 12 m/s is explodes in to two fragments with masses of 10 kg and 5kg. The velocity of the larger fragment is 25m/sec and forms an angle of 330^o with the horizontal. Find the magnitude and direction of the velocity of the smaller fragment.
Solution: The given situation looks like the figure below

$$\begin{pmatrix}\text{Total momentum}\\ \text{before explosion}\end{pmatrix} = \begin{pmatrix}\text{Total Momentum}\\ \text{after explosion}\end{pmatrix}$$

$$(P_{tot})_{\text{before}} = (P_{\text{tot}})_{\text{after}}$$

$$\Rightarrow (15\times 12)\vec{i} = (250\cos 30 + 5 v\cos\theta)\vec{i} + (-250\sin 30 + 5v\sin\theta)\vec{j}$$

$$0 \Rightarrow 5v\sin\theta = 125$$

$$\Rightarrow v\sin\theta = 25$$

$$\Rightarrow 180 = 216.5 + 5v\cos\theta \Rightarrow v\cos\theta = -7.3L \quad (2)$$

$$(1) \div (2) \Rightarrow \tan\theta = 25/-7.3 = -3.4247 \Rightarrow \theta = -73.72^0 =$$

tan is $-ve$ in the 4^{th} or 2^{nd} quadrant for the prblem we take the angle in the 2^{nd}

$\therefore$ using the value of θ in (1)

$$v = \frac{25}{\sin 6} = \frac{25}{\sin 106.28} = 26 m/s$$

Question 4:
A 4N weight rests on a smooth horizontal plane it is struck with a 2N blow that lasts 0.02 sec. Three seconds after the start of the first blos a second blos of -2N is delivered. This lasts for 0.01 sec. What will be the speed of the body after 4 sec?

Solution:
The forces in this problem are
For any t>3.01 sec is the sum of the two areas

$$\text{i.e} \sum^{1} \mathrm{J} = (2\times 0.02)+(0.01\times(-2)) = 0.02N - \sec$$

$$\Rightarrow 0.02N.\sec = \frac{4}{9.8}(v-0) \Rightarrow v = 0.049m/s$$

Question 5:
The angular speed of a helicopter blade increases from 1 rad/s to 64 rad/s in 3 seconds with constant angular acceleration. What angle has the blade turned through in this time? What is the angular acceleration?

Solution -:
The average angular acceleration is given by:

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{64\,\text{rad/s} - 1\,\text{rad/s}}{3\text{s}} = 21\,\text{rad/s}$$

To get the angular displacement, use

$$\theta = \omega_o t + \frac{1}{2}\alpha t^2 = 98\text{rad} \;\; ; \;\; 15\text{revolutions.}$$

Question 6:
On a bicycle, the gears next to the petal are a radius r_1, and on the back wheel, they are a radius r_2. The wheel is a radius r_w Relate the linear speed v of the bicycle to the angular speed at which you pedal.

Solution -:
A bike is built so that the front gears go at the same angular speed as the petals, while the back gears go at the same angular speed as the wheel when you are pedaling. (If you stop pedaling, the gears disengage from the wheel.) The chain connects the front gears to the back gears, so the chain must have the same *linea*r velocity at both the front and the back gears. Call this linear velocity v_{chain} then we have

$$v_{chain} = \omega_{petal} r_1$$

For the front gear, and

$$v_{chain} = \omega_{wheel} r_2$$

Combining the two means that $\omega_{petal} r_1 = \omega_{wheel} r_2$ now we need to get the speed of the bike. The linear speed of the bike is related to the angular speed of the wheels by:

$$v = \omega_{wheel} r_w$$

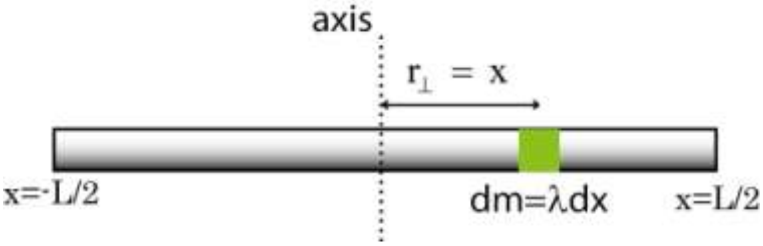
Thus

$$v = \omega_{petal} \frac{r_w r_1}{r_2}$$

Changing the gears on the bike changes the ratio r_1/r_2 changing to a smaller gear in the front, or a larger one in back, (i.e. decreasing r_1/r_2) makes it easier to petal. The reason is for a fixed angular speed ω_{petal} it makes v smaller. The lower v, the less work you're doing.

Question 7:

Consider a thin uniform rod of length and mass . Calculate the moment of inertia about an axis perpendicular to the rod that passes through the center of mass of the rod. A sketch of the rod, volume element, and axis is shown in below.



Solution:

Choose Cartesian coordinates, with the origin at the center of mass of the rod, which is midway between the endpoints since the rod is uniform. Choose the $x-axis$ to lie along the length of the rod, with the positive x-direction to the right, as in the figure.

Identify an infinitesimal mass element $dm = \lambda dx$, located at a displacement x from the center of the rod, where the mass per unit length $\lambda = m/L$ is a constant, as we have assumed the rod to be uniform.

When the rod rotates about an axis perpendicular to the rod that passes through the center of mass of the rod, the element traces out a circle of radius $r_{\perp} = x$ We add together the contributions from each infinitesimal element as we go from $x = -L/2$ to $x = L/2$. The integral is then

$$I_{cm} = \int_{body} (r_{\perp})^2 dm = \lambda \int_{-L/2}^{L/2} (x^2)\, dx = \lambda \left. \frac{x^3}{3} \right|_{-L/2}^{L/2}$$

$$= \frac{m}{L}\frac{(L/2)^3}{3} - \frac{m}{L}\frac{(-L/2)^3}{3} = \frac{1}{12} mL^2$$

By using a constant mass per unit length along the rod, we need not consider variations in the mass density in any direction other than the x - axis. We also assume that the width is the rod is negligible. (Technically we should treat the rod as a rectangle in the $x-y$ plane if the axis is along the z axis. The calculation of the moment of inertia under this assumption would be more complicated.)

Question 8:

A non-homogenous sphere of radius *R* has the following density variation:

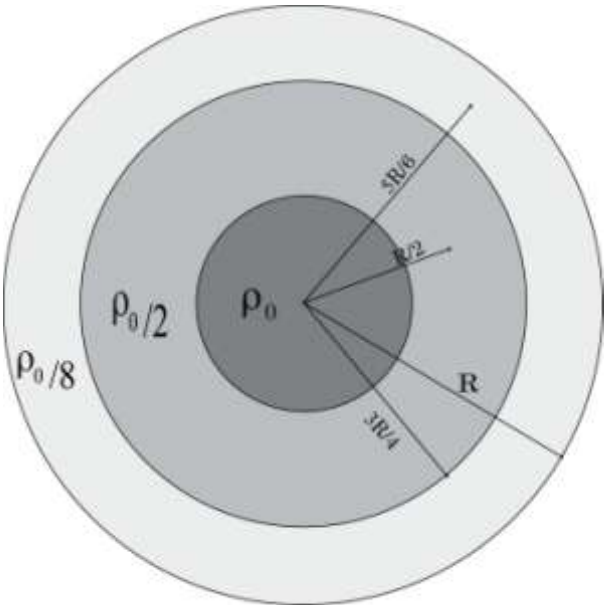
$$p = \rho_0 \text{ for } : r < \frac{R}{3}$$

$$= \frac{\rho_0}{2} \text{ for } : \frac{R}{3} < r \le \frac{3R}{4}$$

$$= \frac{\rho_0}{8} \text{ for } : \frac{R}{3} < r \le R$$

What is the gravitational field due to the sphere at $r = \frac{R}{4}, \ \frac{R}{2}, \ \frac{5R}{6}$ and 2R

Solution:



$$g = \frac{Gm}{r^2} = \frac{GpV}{r^2}$$

$$\therefore g\left(R/4\right) = \frac{G\rho_0 \frac{4}{3}\pi\left(R/4\right)^3}{\left(R/4\right)^2} = \frac{\pi}{3}G\rho_0 R$$

$$g\left(\frac{R}{2}\right) = G\rho_0\frac{4}{3}\pi\left[\frac{\left(R/3\right)^3 + \frac{1}{2}\left[\left(R/2\right)^3 - \left(R/2\right)^3\right]}{(R/2)^2}\right] =$$

$$= G\rho_0\left(4/3\right)\pi R\left[\frac{1}{2}(1/3)^2 + \frac{1}{2}(1/2)^3\right]\times 4$$

$$= G\rho_0\pi R\frac{4}{3}\times 2\left[1/27 + 1/8\right] \quad = 0.432\pi G\rho_0 R$$

$$g\left(\frac{5R}{6}\right) = G\rho_0\frac{4}{3}\pi\left[\frac{\left(R/3\right)^3 + \frac{1}{2}\left[\left(3R/4\right)^3 - \left(R/3\right)^3\right] + \frac{1}{8}\left[\left(5R/6\right)^3 - \left(3R/4\right)^3\right]}{(R/2)^2}\right]$$

$$= 0.48\pi G\rho_0 R$$

Similarly g(2R)= $G\rho_0\left(4/3\right)\pi R\left[\left(1/3\right)^3 + \frac{1}{2}(3/4)^3 - \left(1/3\right)^3\right] + \frac{1}{8}\left[1 - \left(3/4\right)^3\right] \div 4$

$$= 0.1\pi G\rho_0 R$$

Question 9:

Prove that there is no gravitational field due to mass of a spherical shell inside

Solution:

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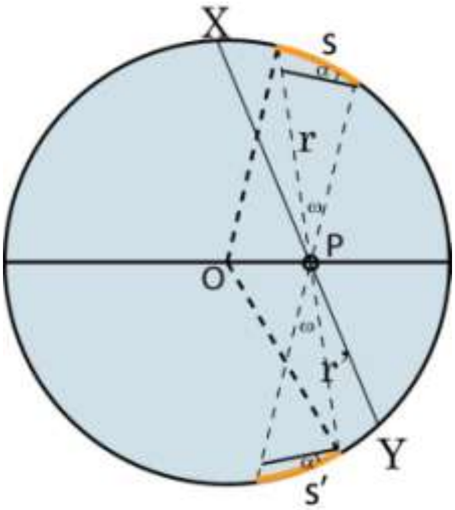


Let p= be any point inside a spherical sheet

$\rho =$ mass per unit area of the shell

Consider two cones, with theis apices at P, intercepting small areas S and S' on the shell as shown draw a plane *x-y* through , 1 the diareter through p

$S = r^2.\omega \qquad S' = r'^2.\omega$

$S \cos\alpha = r^2\omega \qquad S \cos\alpha = r'^2 w$

mass of S = $r^2\omega\rho / \cos\alpha$

mass of S = $r'^2 \omega\rho / \cos\alpha$

$\therefore$ intensity at p due to S= $\frac{\mathrm{r}^2\omega\rho}{\cos\alpha r^2} G = \frac{G\omega\rho}{\cos\alpha}$

and

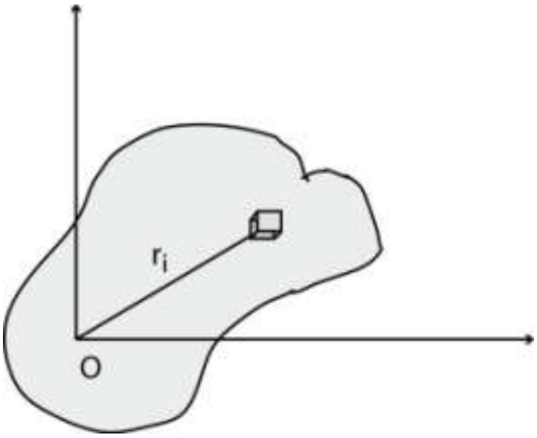
intensity at P due to S' = $\frac{\mathrm{r}'^2\omega\rho}{\cos\alpha r'^2} G = \frac{G\omega\rho}{\cos\alpha}$

These two intensities at P being equal and opposite, their resultant is zero. Similar is the care for all other, pairs of comes on opposite sides of *xy* in to which the shell may be divided so that the resultant intensity or field at p due to the whole shell zero.

Question 10:

For a body of mass *M* , and pivoted at o as shown, show that the gravitational torque acts on it as if the entire mass is concentrated at the center of mass.

Solution



Let us consider the body as being made up of a large number of point masses and one such mass be mi

$$\tau = \sum \tau_i = \sum \vec{r}_i \times m_i \vec{g} = (\sum m_i r_i) \times \vec{g} \quad \because g \text{ is constant}$$

By defintion of center of mass

$$\sum m_i \vec{r}_i = \vec{R}_{cm} \sum m_i = \vec{R}_{cm} M$$

$\vec{R}_{cm} =$ the position vector of the center of mass

Using the above two expressions it follows.

$$\tau = \vec{R}_{cm} \times (m\vec{g})$$

Thus we find that the torque gets as if the entire mass M of the body were conc at its cm.