Mechanics Formulae

Centres of mass

For uniform bodies:

Triangular lamina: $\frac{2}{3}$ along median from vertex

Circular arc, radius r, angle at centre 2α : $\dfrac{r\sin\alpha}{\alpha}$ from centre

Sector of circle, radius r, angle at centre 2α : $\dfrac{2r\sin\alpha}{3\alpha}$ from centre

Motion in a circle

Transverse velocity: $v = r\dot{\theta}$

Transverse acceleration: $\dot{v} = r\ddot{\theta}$

Radial acceleration: $-r\dot{\theta}^2 = -\dfrac{v^2}{r}$

Centres of mass

For uniform bodies:

Solid hemisphere, radius r: $\frac{3}{8}r$ from centre

Hemispherical shell, radius r: $\frac{1}{2}r$ from centre

Solid cone or pyramid of height h: $\frac{1}{4}h$ above the base on the line from centre of base to vertex

Conical shell of height h: $\frac{1}{3}h$ above the base on the line from centre of base to vertex

Universal law of gravitation

$$\text{Force} = \frac{Gm_1m_2}{d^2}$$

Moments of inertia

For uniform bodies of mass m:

Thin rod, length $2l$, about perpendicular axis through centre: $\frac{1}{3}ml^2$

Rectangular lamina about axis in plane bisecting edges of length $2l$: $\frac{1}{3}ml^2$

Thin rod, length $2l$, about perpendicular axis through end: $\frac{4}{3}ml^2$

Rectangular lamina about edge perpendicular to edges of length $2l$: $\frac{4}{3}ml^2$

Rectangular lamina, sides $2a$ and $2b$, about perpendicular axis through centre: $\frac{1}{3}m(a^2+b^2)$

Hoop or cylindrical shell of radius r about axis through centre: mr^2

Hoop of radius r about a diameter: $\frac{1}{2}mr^2$

Disc or solid cylinder of radius r about axis through centre: $\frac{1}{2}mr^2$

Disc of radius r about a diameter: $\frac{1}{4}mr^2$

Solid sphere, radius r, about diameter: $\frac{2}{5}mr^2$

Spherical shell of radius r about a diameter: $\frac{2}{3}mr^2$

Parallel axes theorem: $I_A = I_G + m(AG)^2$

Perpendicular axes theorem: $I_z = I_x + I_y$ (for a lamina in the x-y plane)

Moments as vectors

The moment about O of $\mathbf{F}$ acting at $\mathbf{r}$ is $\mathbf{r} \times \mathbf{F}$