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Matrices and Determinants

Question 1: **Solve for *x, y* and *z***

$$\begin{cases} x-2y=6-4z \\ x+13z=6-y \\ -2x+6y-z=-1 \end{cases} \quad \text{or} \quad \begin{cases} x-2y-4z=6 \\ x+y+13z=6 \\ -2x+6y-z=-1 \end{cases}$$

Augmented Matrix:

$$\begin{bmatrix} 1 & -2 & 4 & 6 \\ 1 & 1 & 13 & 6 \\ -2 & 6 & -1 & -1 \end{bmatrix}$$

Multiply row 1 by –1 and add with row 2 and get your new row 2

$$\begin{bmatrix} 1 & -2 & 4 & 6 \\ 0 & 3 & 9 & 0 \\ -2 & 6 & -1 & -1 \end{bmatrix};\ R_2' = R_2 - 1R_1$$

Multiply 2 with row 1 and add with row 3 and get your new row 3

$$\begin{bmatrix} 1 & -2 & 4 & 6 \\ 0 & 3 & 9 & 0 \\ 0 & 2 & 7 & 11 \end{bmatrix};\ R_3' = R_3 + 2R_1$$

Multiply row 2 by 1/3 gives you following

$$\begin{bmatrix} 1 & -2 & 4 & 6 \\ 0 & 1 & 3 & 0 \\ 0 & 2 & 7 & 11 \end{bmatrix};\ R_2' = \frac{1}{3}R_2$$

Multiply by -2 with row 2 and add with row 3 and get your row 3

$$\begin{bmatrix} 1 & -2 & 4 & 6 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 11 \end{bmatrix};\ R_3' = R_3 - 2R_2$$

Now by backward substitution to solve for variables:

$$z = 11, \qquad y + 3(11) = 0 \qquad x - 2(-6) + 4(11) = 6$$

$y = -33 \qquad\qquad x = -104$

Now practice: $\begin{bmatrix} 2 & 1 & -1 & 2 \\ 1 & 3 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$ ☺ Write the system of equations and solve. Ans (2, -1, 1)

Question 2:
Below are three row-reduced echelon forms for matrices of certain linear systems. For each matrix, tell **how many solutions** the system has. **Explain.** If there are any, **find the solutions.** If there are infinitely many solutions, **find the general formula and two particular solutions.**

a. $\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 5 \end{bmatrix}$ The system has unique solution with *x=3, y=-2, z=5*

b. $\begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ The system does not have solution

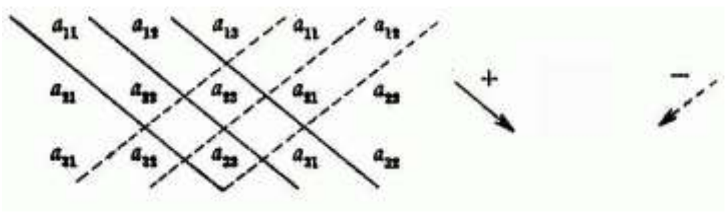
c. $\begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ The system has many solutions, it is dependent and consistent.

The general solutions are $\begin{cases} y + 3z = 0 \\ y = -3z \end{cases}$ and $\begin{cases} x - 2z = 0 \\ x = 2z \end{cases}$

Question 1:

Sarrus Rule. In order to compute

$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$



Note: Sarrus rule is only applicable if the determinant is of order 3 by 3.

Example: Use Sarrus rule to find the value of $A = \begin{vmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{vmatrix}$

12 + 4 + 48 = 64

$\begin{matrix} 2 & 3 & 4 & 2 & 3 \\ 4 & 3 & 1 & 4 & 3 \\ 1 & 2 & 4 & 1 & 2 \end{matrix} \quad \Rightarrow A = 59 - 64 = -5$

24 + 3 + 32 = 59

Question 4:

Finding inverse of a matrix:

Now let's learn how to find inverse of a matrix. There are different methods to find inverse matrix.

Method 1. Use Shortcut for 2 by 2 matrix

Let $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $A^{-1}=\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Example:

$$A=\begin{bmatrix} 3 & 2 \\ 5 & -3 \end{bmatrix} \text{then } A^{-1}=\frac{1}{3(-3)-5(2)}\begin{bmatrix} -3 & -2 \\ -5 & 3 \end{bmatrix}=\begin{bmatrix} \frac{3}{19} & \frac{2}{19} \\ \frac{5}{19} & \frac{-3}{19} \end{bmatrix}$$

Method 2 (Optional). Use Gauss-Jordan elimination to transform [A | I] into [I | A^{-1}].

Example: Consider a matrix $A=\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and write the following

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{array}\right] \to \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -2 & -3 & 1 \end{array}\right]$$

$$\to \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & 3/2 & -1/2 \end{array}\right]$$

$$\to \left[\begin{array}{cc|cc} 1 & 0 & -2 & 1 \\ 0 & 1 & 3/2 & -1/2 \end{array}\right]$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix}$$

Method 3: **Adjoint method**

$A^{-1} = \frac{1}{\det A}$(adjoint of A) or $A^{-1} = \frac{1}{\det A}$ (cofactor matrix of A)^T

Let $A=\begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$, *then* $|A|=-5$ and $A^{-1}=\frac{1}{-5}\begin{bmatrix} 10 & -4 & -9 \\ -15 & 4 & 14 \\ 5 & -1 & -6 \end{bmatrix}$

Now we know how to find inverse, let's go back to solution of system of equations:

Example 1: (continued) given system is $\begin{cases} 3x+2y=-1 \\ 5x-3y=11 \end{cases}$

If we write in matrix form then we get the following,

$$A=\begin{bmatrix} 3 & 2 \\ 5 & -3 \end{bmatrix} \qquad X=\begin{bmatrix} x \\ y \end{bmatrix} \qquad B=\begin{bmatrix} -1 \\ 11 \end{bmatrix}$$

If we do $AX=B$, we get the given system and we can rewrite

$$AX=B$$
$$\Rightarrow X=A^{-1}B$$

We have seen that $A=\begin{bmatrix} 3 & 2 \\ 5 & -3 \end{bmatrix}$ then $A^{-1}=\frac{1}{3(-3)-5(2)}\begin{bmatrix} -3 & -2 \\ -5 & 3 \end{bmatrix}=\begin{bmatrix} \frac{3}{19} & \frac{2}{19} \\ \frac{5}{19} & \frac{-3}{19} \end{bmatrix}$

Now $\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} \frac{3}{19} & \frac{2}{19} \\ \frac{5}{19} & \frac{-3}{19} \end{bmatrix}\times\begin{bmatrix} -1 \\ 11 \end{bmatrix}=\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ so $x=1$ and $y=-2$

Question 5:

Use Cramer's Rule to solve the system:

$$4x - y + z = -5$$

$$2x + 2y + 3z = 10$$

$$5x - 2y + 6z = 1$$

Solution. We begin by setting up four determinants:

$$D, D_x, D_y, \text{ and } D_z:$$

D consists of the coefficients of *x*, *y*, and *z* from the three equations

$$D=\begin{vmatrix} 4 & -1 & 1 \\ 2 & 2 & 3 \\ 5 & -2 & 6 \end{vmatrix} \qquad D_x=\begin{vmatrix} -5 & -1 & 1 \\ 10 & 2 & 3 \\ 1 & -2 & 6 \end{vmatrix} \qquad D_y=\begin{vmatrix} 4 & -5 & 1 \\ 2 & 10 & 3 \\ 5 & 1 & 6 \end{vmatrix} \qquad D_z=\begin{vmatrix} 4 & -1 & -5 \\ 2 & 2 & 10 \\ 5 & -2 & 1 \end{vmatrix}$$

D_x is obtained by replacing the x-coefficients in the first column of D with the constants from the right sides of the equations.

D_y is obtained by replacing the y-coefficients in the second column of D with the constants from the right sides of the equations.

D_z is obtained by replacing the z-coefficients in the third column of D with the constants from the right sides of the equations.

Next, we evaluate the four determinants:

$$D = \begin{vmatrix} 4 & -1 & 1 \\ 2 & 2 & 3 \\ 5 & -2 & 6 \end{vmatrix} = 4\begin{vmatrix} 2 & 3 \\ -2 & 6 \end{vmatrix} - (-1)\begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} + 1\begin{vmatrix} 2 & 2 \\ 5 & -2 \end{vmatrix}$$

= 4(12 – (-6)) + 1(12 – 15) + 1(-4 – 10)

= 4(18) + 1(-3) + 1(-14)

= 72 – 3 – 14

= 55

$$D_x = \begin{vmatrix} -5 & -1 & 1 \\ 10 & 2 & 3 \\ 1 & -2 & 6 \end{vmatrix} = -5\begin{vmatrix} 2 & 3 \\ -2 & 6 \end{vmatrix} - (-1)\begin{vmatrix} 10 & 3 \\ 1 & 6 \end{vmatrix} + 1\begin{vmatrix} 10 & 2 \\ 1 & -2 \end{vmatrix}$$

= -5(12 – (-6)) + 1(60 – 3) + 1(-20 – 2)

= -5(18)+1(57) + 1(-22)

= -90 + 57 – 22

= -55

$$D_y = \begin{vmatrix} 4 & -5 & 1 \\ 2 & 10 & 3 \\ 5 & 1 & 6 \end{vmatrix} = 4\begin{vmatrix} 10 & 3 \\ 1 & 6 \end{vmatrix} - (-5)\begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} + 1\begin{vmatrix} 2 & 10 \\ 5 & 1 \end{vmatrix}$$

= 4(60 – 3) + 5(12 – 15) + 1(2 – 50)

= 4(57) + 5(-3) + 1(-48)

= 228 - 15 – 48

= 165

$$D_z = \begin{vmatrix} 4 & -1 & -5 \\ 2 & 2 & 10 \\ 5 & -2 & 1 \end{vmatrix} = 4\begin{vmatrix} 2 & 10 \\ -2 & 1 \end{vmatrix} - (-1)\begin{vmatrix} 2 & 10 \\ 5 & 1 \end{vmatrix} + (-5)\begin{vmatrix} 2 & 2 \\ 5 & -2 \end{vmatrix}$$

= 4(2 – (-20)) + 1(2 – 50) – 5(-4 – 10)

= 4(22) + 1(-48) – 5(-14)

= 88 – 48 + 70

= 110

Substitute these four values into the formula from Cramer's Rule:

$$x = \frac{D_x}{D} = \frac{-55}{55} = -1 \qquad y = \frac{D_y}{D} = \frac{165}{55} = 3 \qquad z = \frac{D_z}{D} = \frac{110}{55} = 2$$

So, the solution is (-1, 3, 2).

Question 6:

Solve the system

$$\begin{aligned} x_1 + x_2 + 3x_3 &= 5 \\ 2x_2 - x_3 &= 1 \\ -3x_1 + 2x_2 + 2x_3 &= 1 \end{aligned}$$

Solution: Now applying the operation $R_3 = r_3 + 3r_1$ we have the following

$$\begin{aligned} x_1 + x_2 + 3x_3 &= 5 \\ 2x_2 - x_3 &= 1 \\ 5x_2 + 11x_3 &= 16 \end{aligned}$$

Applying $R_2 = 1/2\,r_2$ we have

$$\begin{aligned} x_1 + x_2 + 3x_3 &= 5 \\ x_2 - .5x_3 &= .5 \\ 5x_2 + 11x_3 &= 16 \end{aligned}$$

And by $R_3 = r_3 - 5r_2$

$$\begin{aligned} x_1 + x_2 + 3x_3 &= 5 \\ x_2 - .5x_3 &= .5 \\ 13.5x_3 &= 13.5 \end{aligned}$$

Finally we the following by applying $R_3 = r_2 / 13.5$

$$\begin{aligned} x_1 + x_2 + 3x_3 &= 5 \\ x_2 - .5x_3 &= .5 \\ x_3 &= 1 \end{aligned}$$

We now have that $x_3 = 1$, and other unknowns can easily be found by backward substitution into second and first equations. We have the solution $(x_1, x_2, x_3) = (1, 1, 1)$. This method is called the Gaussian Elimination method.

Question 7:

$$A = \begin{bmatrix} 4 & 10 \\ -2 & -3 \\ 6 & 9 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 3 & 0 \\ 0 & 1 & 5 \end{bmatrix} \quad C = \begin{bmatrix} 5 & 0 \\ -1 & 3 \\ 4 & 7 \end{bmatrix}$$

$$D = \begin{bmatrix} 6 \\ 1 \\ 0 \end{bmatrix} \quad E = \begin{bmatrix} 1 & 3 & -4 \end{bmatrix} \quad F = \begin{bmatrix} -1 & 2 \\ 8 & 7 \end{bmatrix} \quad G = \begin{bmatrix} 2 & 5 \\ 1 & 6 \end{bmatrix}$$

Order of above matrices:
A is a matrix 3 by 2, B is 3 by 3, C is 3 by 2, D is 3 by 1, E is 1 by 3, F and G are 2 by 2 matrices.

Find the following if possible

1. -B, 2. 3A-2C, 3. F+3G, 4. 2B-5C

Answers:

$$-B = \begin{bmatrix} -1 & -3 & -2 \\ -2 & -3 & 0 \\ 0 & -1 & -5 \end{bmatrix}$$

$$\text{3A-2C} = 3\begin{bmatrix} 4 & 10 \\ -2 & -3 \\ 6 & 9 \end{bmatrix} - 2\begin{bmatrix} 5 & 0 \\ -1 & 3 \\ 4 & 7 \end{bmatrix} = \begin{bmatrix} 2 & 30 \\ -4 & -15 \\ 10 & 13 \end{bmatrix}$$

$$F + 3G = \begin{bmatrix} -1 & 2 \\ 8 & 7 \end{bmatrix} + 3\begin{bmatrix} 2 & 5 \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} 5 & 17 \\ 11 & 25 \end{bmatrix}$$

2B-5C is not possible