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Markov's Chain

Question 1:

Consider the Markov chain that has the following (one-step) transition matrix.

$$P = \begin{array}{c} \text{State} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0.2 & 0.5 & 0.3 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0.2 & 0 & 0.1 & 0.7 \\ 0 & 1 & 0 & 0 & 0 \\ 0.8 & 0.1 & 0 & 0.1 & 0 \end{bmatrix} \end{array}$$

(a) Determine the classes of this Markov chain and, for each class, determine whether it is recurrent or transient.

(b) For each of the classes identified in part (*a*), determine the period of the states in that class.

Solution:

(a) States 1 and 3 are accessible from each other (p_{31} = 1 and p_{13} = 1), but no other states are accessible from these states (p_{1j} = 0 and p_{3j} = 0 for j = 0, 2, 4). Therefore, states 1 and 3 communicate and form one class of the Markov chain. Upon entering either state, the process will return to that state in two steps, so {1, 3} is a recurrent class.

State 0 is accessible from state 4 (p_{40} = 0.8), state 2 is accessible from state 0 (p_{02} = 0.5), and state 4 is accessible from state 2 (p_{24} = 0.7), so each of these states is accessible from each of these other states. Therefore, states 0, 2, and 4 communicate and form a second class of the Markov chain. The process can move from any of these states to state 1 or state 3, in which case the process never would return to states 0, 2, and 4 again. Therefore, {0, 2, 4} is a transient class.

(b) We calculate $P^{(2)}$ and $P^{(3)}$.

$$P^{(2)} = P*P = \begin{bmatrix} 0 & 0.4 & 0 & 0.25 & 0.35 \\ 0 & 1 & 0 & 0 & 0 \\ 0.56 & 0.17 & 0 & 0.27 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0.26 & 0.4 & 0.34 & 0 \end{bmatrix}.$$

$$P^{(3)} = P^{(2)}*P = \begin{bmatrix} 0.28 & 0.285 & 0 & 0.435 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0.382 & 0.28 & 0.338 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 0.3 & 0.28 \end{bmatrix}.$$

Since $p_{11} = p_{33} = 0$ and $p^{(2)}_{11} = p^{(2)}_{33} = 1$, the class {1, 3} has period 2.

Now note that $p_{00} = p_{22} = p_{44} = 0$, $p^{(2)}_{00} = p^{(2)}_{22} = p^{(2)}_{44} = 0$, and $p^{(3)}_{00} > 0$, $p^{(3)}_{22} > 0$, $p^{(3)}_{44} > 0$. This indicates that the class {0, 2, 4} has period 3.

Question 2:

A soap company specializes in a luxury type of bath soap. The sales of this soap fluctuate between two levels — "Low" and "High"— depending upon two factors: (1) whether they advertise, and (2) the advertising and marketing of new products being done by competitors. The second factor is out of the company's control, but it is trying to determine what its own advertising policy should be. For example, the marketing manager's proposal is to advertise when sales are low but not to advertise when sales are high. Advertising in any quarter of a year has its primary impact on sales in the *following* quarter. Therefore, at the beginning of each quarter, the needed information is available to forecast accurately whether sales will be low or high that quarter and to decide whether to advertise that quarter.

The cost of advertising is $1 million for each quarter of a year in which it is done. When advertising is done during a quarter, the probability of having high sales the next quarter is 1/2 or 3/4, depending upon whether the current quarter's sales are low or high. These probabilities go down to 1/4 or1/2 when advertising is not done during the current quarter. The company's quarterly profits (excluding advertising costs) are $4 million when sales are high but only $2 million when sales are low. (Hereafter, use units of million of dollars.)

A) Construct the (one-step) transition matrix for each of the following advertising strategies: (*i*) never advertise, (*ii*) always advertise, (*iii*) follow the marketing manager's proposal.
B) Find the long run expected average profit (including a deduction for advertising costs) per quarter for each of the three advertising strategies in part (A). Which of these strategies is best according to this measure of performance?

Solution:

Let state 0 indicate the "Low" level of sales and state 1 indicate the "High" level of sales during the current quarter, where each transition of the process goes from one quarter to the next.

(i) The one-step transition matrix for the "never advertise" strategy is

State 0 1

$$\mathbf{P} = \begin{matrix} 0 \\ 1 \end{matrix} \begin{bmatrix} 3/4 & 1/4 \\ 1/2 & 1/2 \end{bmatrix}.$$

(ii) The one-step transition matrix for the "always advertise" strategy is

State 0 1

$$\mathbf{P} = \begin{matrix} 0 \\ 1 \end{matrix} \begin{bmatrix} 1/2 & 1/2 \\ 1/4 & 3/4 \end{bmatrix}.$$

(iii) The one-step transition matrix for the marketing manager's proposal is

State 0 1

$$\mathbf{P} = \begin{matrix} 0 \\ 1 \end{matrix} \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}.$$

B) Recall that the company's quarterly profits (excluding advertising costs) are \$4 million when sales are high (State = 1) and \$2 million when sales are low (State = 0). Therefore, in units of millions of dollars, the long run expected average cost (excluding advertising costs) is $2\pi_0 + 4\pi_1$. To take advertising costs (\$1 million) into account, we need to subtract 1 from each coefficient where advertising is done. This leads to the calculations shown below.

For (i) the "never advertise" strategy, the long-run expected average profit is

profit = 2(2/3) + 4(1/3) = \$ 8/3 million.

For (ii) the "always advertise" strategy, the long-run expected average profit is

profit = 2(1/3) + 4(2/3) -1 = \$ 7/3 million.

For (iii) the marketing manager's proposal, the long-run expected average profit is

profit = (2-1)(1/2) + 4(1/2) = \$ 5/2 million.

Therefore, when the objective is to maximize the long run expected average profit, the best strategy is "never advertise".