

Laplace Transforms Formulae

If $y(t)$ is a function defined for $t \geq 0$, the Laplace transform $\bar{y}(s)$ is defined by the equation

$$\bar{y}(s) = \mathcal{L}\{y(t)\} = \int_0^\infty e^{-st} y(t) dt$$

| Function $y(t)$ ($t > 0$) | Transform $\bar{y}(s)$ | |
|--|---|---------------------|
| $\delta(t)$ | 1 | Delta function |
| $\theta(t)$ | $\frac{1}{s}$ | Unit step function |
| t^n | $\frac{n!}{s^{n+1}}$ | |
| $t^{\frac{1}{2}}$ | $\frac{1}{2} \sqrt{\frac{\pi}{s^3}}$ | |
| $t^{-\frac{1}{2}}$ | $\sqrt{\frac{\pi}{s}}$ | |
| e^{-at} | $\frac{1}{(s+a)}$ | |
| $\sin \omega t$ | $\frac{\omega}{(s^2 + \omega^2)}$ | |
| $\cos \omega t$ | $\frac{s}{(s^2 + \omega^2)}$ | |
| $\sinh \omega t$ | $\frac{\omega}{(s^2 - \omega^2)}$ | |
| $\cosh \omega t$ | $\frac{s}{(s^2 - \omega^2)}$ | |
| $e^{-at} y(t)$ | $\bar{y}(s+a)$ | |
| $y(t-\tau) \theta(t-\tau)$ | $e^{-s\tau} \bar{y}(s)$ | |
| $ty(t)$ | $-\frac{d\bar{y}}{ds}$ | |
| $\frac{dy}{dt}$ | $s\bar{y}(s) - y(0)$ | |
| $\frac{d^n y}{dt^n}$ | $s^n \bar{y}(s) - s^{n-1} y(0) - s^{n-2} \left[\frac{dy}{dt} \right]_0 - \cdots - \left[\frac{d^{n-1} y}{dt^{n-1}} \right]_0$ | |
| $\int_0^t y(\tau) d\tau$ | $\frac{\bar{y}(s)}{s}$ | |
| $\begin{cases} \int_0^t x(\tau) y(t-\tau) d\tau \\ \int_0^t x(t-\tau) y(\tau) d\tau \end{cases}$ | $\bar{x}(s) \bar{y}(s)$ | Convolution theorem |

[Note that if $y(t) = 0$ for $t < 0$ then the Fourier transform of $y(t)$ is $\hat{y}(\omega) = \bar{y}(i\omega)$.]