Math Assignment Experts is a leading provider of online Math help. Our experts have prepared sample assignments to demonstrate the quality of solution we provide. If you are looking for mathematics help then share your requirements at info@mathassignmentexperts.com

Coordinate Geometry Problems

Question 1

$A(2,3),\ B(-1,-2),\ C(-9,10)$

$\vec{AB} = \begin{pmatrix}-3\\-5\end{pmatrix}$.

$\vec{BC} = \begin{pmatrix}-8\\12\end{pmatrix}$

$\vec{CA} = \begin{pmatrix}11\\-7\end{pmatrix}$

$AB = \sqrt{34}$

$BC = \sqrt{208}$

$CA = \sqrt{170}$

So by Hero, area

$$= \frac{1}{16}\sqrt{(\sqrt{34}+\sqrt{208}+\sqrt{170})(\sqrt{34}+\sqrt{208}-\sqrt{170})(\sqrt{34}-\sqrt{208}+\sqrt{170})(-\sqrt{34}+\sqrt{208}+\sqrt{170})}$$

$$= \frac{1}{16}\sqrt{[34+208+2\sqrt{34}\sqrt{208}-170]\times[170-(34+208-2\sqrt{34}\sqrt{208})]}$$

$$= \frac{1}{16}\sqrt{[2\sqrt{34}\sqrt{208}-72]\times[2\sqrt{34}\sqrt{208}+72]}$$

$$= \frac{1}{16}\sqrt{[4\times34\times208-72^2]}$$

$$= \frac{1}{16}\sqrt{[16(17\times104-18^2)]}$$

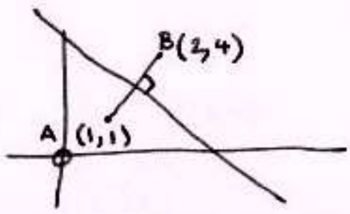
$$= \frac{1}{16}\sqrt{[16(1768-324)]}$$

$$= \frac{1}{4}\sqrt{1444}$$

$$= \frac{1}{2}\sqrt{361}$$

$$= \frac{19}{2} \text{ or } 9\tfrac{1}{2} \text{ units}^2$$

Question 2



Midpt of AB is $(1\tfrac{1}{2}, 2\tfrac{1}{2})$

$m_{AB} = \frac{4-1}{2-1}$

$= 3$

$m_{\perp AB} = -\frac{1}{3}$

Equation of locus is

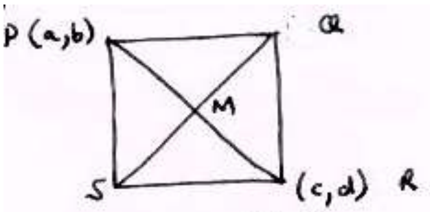
$y - 2\tfrac{1}{2} = -\tfrac{1}{3}(x - 1\tfrac{1}{2})$

$\Rightarrow 6y - 15 = -2x + 3$

$\Rightarrow 0 = 2x + 6y - 18$

$\Rightarrow 0 = x + 3y - 9$

Question 3



M is $\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$

$$\overrightarrow{PR} = \begin{pmatrix} c-a \\ d-b \end{pmatrix}$$

$$\Rightarrow \overrightarrow{SQ} = \begin{pmatrix} b-d \\ c-a \end{pmatrix}$$

$$\underline{m} = \begin{pmatrix} \frac{a+c}{2} \\ \frac{b+d}{2} \end{pmatrix}$$

$$\Rightarrow \underline{q} = \underline{m} + \tfrac{1}{2}\overleftrightarrow{SQ}$$

$$= \begin{pmatrix} \frac{a+c}{2} \\ \frac{b+d}{2} \end{pmatrix} + \begin{pmatrix} \frac{b-d}{2} \\ \frac{c-a}{2} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{a+b+c-d}{2} \\ \frac{b+c+d-a}{2} \end{pmatrix}$$

and $\underline{s} = \begin{pmatrix} \frac{a+c+d-b}{2} \\ \frac{a+b+d-c}{2} \end{pmatrix}$

So the coordinates are $\left(\frac{a+b+c-d}{2}, \frac{b+c+d-a}{2}\right)$ and $\left(\frac{a+c+d-b}{2}, \frac{a+b+d-c}{2}\right)$

Question 4

$$\left(\frac{2k}{k^2+1}\right)^2 + \left(\frac{k^2-1}{k^2+1}\right)^2$$

$$= \frac{4k^2 + k^4-2k^2+1}{(k^2+1)^2}$$

$$= \frac{k^4+2k^2+1}{(k^2+1)^2}$$

$$= 1$$

Hence the point is always distant 1 from O. QED.

For k an integer, can two values give the same point? Suppose

$$\frac{2k}{k^2+1} = \frac{2m}{m^2+1}$$

$$\Rightarrow 2k(m^2+1) = 2m(k^2+1)$$

$$\Rightarrow 2km^2+2k-2k^2m-2m = 0$$

$$\Rightarrow 2km(m-k)+2(k-m) = 0$$

$$\Rightarrow 2(km-1)(m-k) = 0$$

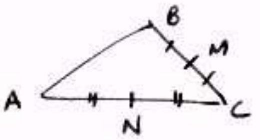
$$\Rightarrow m = k$$

$$\text{or } k = \frac{1}{m}$$

Thus if m and k are different and integral, neither is satisfied, so different values of k give different points, and hence infinitely many rational points which are distinct.

Question 5

$A(1,4)$, $B(7,5)$, $C(1,8)$



$M(4, 6\frac{1}{2})$, $N(1,6)$

$m_{BC} = \frac{8-5}{1-7}$

$= -\frac{1}{2}$

$\Rightarrow m_{\perp BC} = 2$

AC's gradient is undefined.

$\perp$ar bisector of AC is

$y = 6$

$\perp$ar bisector of BC is

$y - 6\frac{1}{2} = 2(x-4)$

$\Rightarrow 2y - 13 = 4x - 16$

$\Rightarrow 0 = 4x - 2y - 3$

When $y = 6$, $x = \frac{15}{4}$.

So centre is $\left(\frac{15}{4}, 6\right)$.

Hence equation is

$\left(x - \frac{15}{4}\right)^2 + (y-6)^2 = \left(\frac{11}{4}\right)^2 + 2^2$

$\Rightarrow x^2 - \frac{15}{2}x + \frac{225}{16} + y^2 - 12y + 36$

$= \frac{121}{16} + 4$

$\Rightarrow x^2 + y^2 - \frac{15}{2}x - 12y$

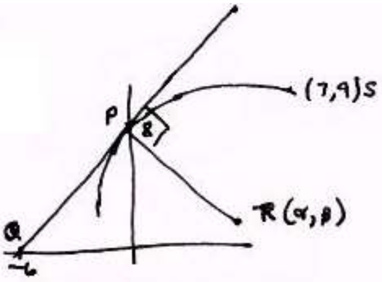
$= -\frac{104}{16} - 32$

$= -6\frac{1}{2} - 32$

$\Rightarrow x^2 + y^2 - \frac{15}{2}x - 12y + \frac{77}{2}$

$= 0$

Question 6



$m_{PQ} = \frac{4}{3}$

$\Rightarrow m_{PR} = -\frac{3}{4}$

Equation of PR is

$y - 8 = -\frac{3}{4}x$

$\Rightarrow y = -\frac{3}{4}x + 8$

So $\beta = -\frac{3}{4}\alpha + 8$

$PR = RS$

$\Rightarrow \alpha^2 + (\beta - 8)^2$

$= (\alpha - 7)^2 + (\beta - 9)^2$

$\Rightarrow \alpha^2 + \beta^2 - 16\beta + 64$

$= \alpha^2 - 14\alpha + 49 + \beta^2 - 18\beta + 81$

$\Rightarrow 0 = -14\alpha - 2\beta + 56$

$= -14\alpha - 2\left(-\frac{3}{4}\alpha + 8\right) + 66$

$= -14\alpha + \frac{3}{2}\alpha - 16 + 66$

$= -\frac{25}{2}\alpha + 50$

$\Rightarrow \alpha = \frac{100}{25}$

$= 4$

$\Rightarrow \beta = -3 + 8$

$= 5$

So circle is

$(x-4)^2 + (y-5)^2$

$= (4-7)^2 + (5-9)^2$

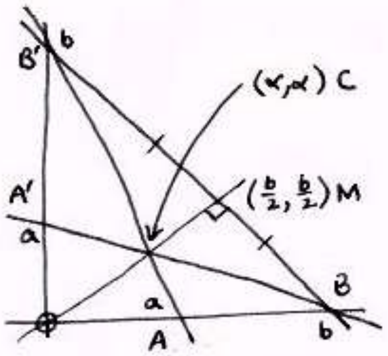
$\Rightarrow x^2 - 16x + 16 + y^2 - 10y + 25$

$= 9 + 16$

$\Rightarrow x^2 - 16x + y^2 - 10y$

$= 16$

Question 7



$m_{CB'} = m_{AC}$

$$\Rightarrow \frac{\alpha - b}{\alpha - 0} = \frac{0-\alpha}{a-\alpha}$$

$$\Rightarrow (\alpha-b)(a-\alpha) = -\alpha^2$$

$$\Rightarrow -\alpha^2 - ba + b\alpha + a\alpha = -\alpha^2$$

$$\Rightarrow \alpha = \frac{ba}{a+b}$$

$$CM^2 = 2\left(\frac{ba}{a+b} - \frac{b}{2}\right)^2$$

$$= 2b^2\left(\frac{a}{a+b} - \frac{1}{2}\right)^2$$

$$= \frac{b^2}{2}\left(\frac{2a-a-b}{a+b}\right)^2$$

$$= \frac{b^2(a-b)^2}{2(a+b)^2}$$

$$\Rightarrow CM = \frac{b(b-a)}{\sqrt{2}(a+b)}$$

(since $b > a$)

So shaded area

$$= \frac{1}{2} . BB' . CM$$

$$= \frac{1}{2} . \sqrt{2}b . \frac{b(b-a)}{\sqrt{2}(a+b)}$$

$$= \frac{b^2(b-a)}{2(a+b)}$$