Complex Variables Formulae

Complex numbers

The complex number $z = x + \mathrm{i}y = r(\cos\theta + \mathrm{i}\sin\theta) = r\,\mathrm{e}^{\mathrm{i}(\theta+2n\pi)}$, where $\mathrm{i}^2 = -1$ and n is an arbitrary integer. The real quantity r is the modulus of z and the angle θ is the argument of z. The complex conjugate of z is $z^* = x - \mathrm{i}y = r(\cos\theta - \mathrm{i}\sin\theta) = r\,\mathrm{e}^{-\mathrm{i}\theta}$; $\quad zz^* = |z|^2 = x^2 + y^2$

De Moivre's theorem

$$(\cos\theta + \mathrm{i}\sin\theta)^n = \mathrm{e}^{\mathrm{i}n\theta} = \cos n\theta + \mathrm{i}\sin n\theta$$

Power series for complex variables.

$$\mathrm{e}^z = 1 + z + \frac{z^2}{2!} + \cdots + \frac{z^n}{n!} + \cdots \qquad \text{convergent for all finite } z$$

$$\sin z = z - \frac{z^3}{3!} + \frac{z^5}{5!} - \cdots \qquad \text{convergent for all finite } z$$

$$\cos z = 1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \cdots \qquad \text{convergent for all finite } z$$

$$\ln(1+z) = z - \frac{z^2}{2} + \frac{z^3}{3} - \cdots \qquad \text{principal value of } \ln(1+z)$$

This last series converges both on and within the circle $|z| = 1$ except at the point $z = -1$.

$$\tan^{-1} z = z - \frac{z^3}{3} + \frac{z^5}{5} - \cdots$$

This last series converges both on and within the circle $|z| = 1$ except at the points $z = \pm\mathrm{i}$.

$$(1+z)^n = 1 + nz + \frac{n(n-1)}{2!}z^2 + \frac{n(n-1)(n-2)}{3!}z^3 + \cdots$$

This last series converges both on and within the circle $|z| = 1$ except at the point $z = -1$.