Algebra

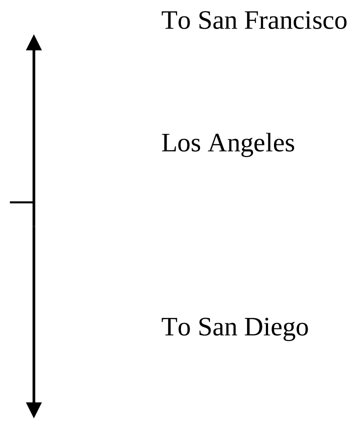
Question 1:

Two trains leave Los Angeles at the same time. Train A travels north. Train B travels south. At the end of two hours they are 180 miles apart. Find the rate of both trains if Train A is traveling 10 miles per hour slower than Train B.

Solution:

Preliminary Steps: On these types of problems it is helpful to draw a diagram and a chart. Also, remember that distance = rate x time. Rate is the same as speed. The other steps are the same.

Drawing and Chart



	Rate	Time	Distance
Train A	x	2 hours	2x
Train B	x + 10	2 hours	2(x +10)

Step 1: Let x = the rate of train A

Step 2: x + 10 = the rate of train B

Step 3: The total distance, 180 miles, equals the distance train A went plus the distance train B went. Distance = rate multiplied by time or D = rt. t = 2 hours

$2x + 2(x + 10) = 180$

Step 4:

$2x + 2(x + 10) = 180$

$2x + 2x + 20 = 180$ (distributive property)

$4x + 20 = 180$ (combine like terms)

$4x = 160$ (subtract 20 from both sides)

$x = 40$ (Divide both sides by 4)

Step 5: Train A's rate was 40 mph. Train B's rate was x + 10 = 50 mph.

Question 2:

Simplify the following expression as much as possible:

$\overline{\overline{A(A+B)} + B\overline{A}}$

Solution:

Using various elementary identities, the expression simplifies as follows:

$$\overline{\overline{A(A+B)} + B\overline{A}} = \overline{\overline{A(A+B)}} * \overline{B * \overline{A}}$$

$$= A(A+B) * (\overline{B} + A)$$

$$= (A + AB)(\overline{B} + A)$$

$$= A(1+B)(\overline{B} + A)$$

$$= A(1)(\overline{B} + A)$$

$$= A(\overline{B} + A)$$

$$= A\overline{B} + AA$$

$$= A\overline{B} + A$$

$$= A(\overline{B} + 1)$$

$$= A(1) = A$$

Question 3:

Find all ordered pairs (A, B) that make the following expression TRUE.

$\overline{\overline{A+B} + \overline{A} * B}$

Solution:

$$\overline{\overline{A+B} + \overline{A} * B} = (\overline{\overline{A+B}})(\overline{\overline{A}B})$$

$$= (A+B)(A+\overline{B})$$

$$= AA + A\overline{B} + BA + B\overline{B}$$

$$= A + A(B+\overline{B}) + 0$$

$$= A + A(1) = A + A = A$$

This yields the solutions (1, 0) and (1, 1). This problem, like most Boolean Algebra problems, could also be solved by drawing a truth table with the following seven column headings: A, B, A+B,

$\overline{A+B}, \overline{A}B, \overline{A+B} + \overline{A}B, \overline{\overline{A+B} + \overline{A}B}$.

Question 4:

Triangle –second angle 20° more than first, third angle twice the first

Solution:

Step 1: Let n = the first angle.

Step 2: Let n + 20 = the second angle

Let 2n = the third angle

Step 3: n + (n + 20) + 2n = 180

Step 4: 4n + 20 = 180

4n = 160

n = 40

Step 5: second angle = n + 20 = 40 + 20 = 60

Third angle = 2n = 2(40) = 80

Answer: 40°, 60°, 80°

Question 5:

John invested $30,000 for one year, part at 2%, part at 3%. After 1 year, he earns $800. Find how much he invested at 2% and 3% respectively

Solution:

Step 1: Let x = amount invested at 2%

Step 2: $30,000 – x = amount invested at 3%

Step 3: I = prt.

Here, interest (I) ($800) will equal interest on amount (x) at 2% (.02) and interest on amount ($30,000 –x) at 3% (.03). t is one year. Hence, x(.02)(1) + (30,000 – x)(.03)(1) = 800

Step 4:

x(.02)(1) + (30,000 – x)(.03)(1) = 800

.02x + 900 - .03x = 800 (applying distributive property)

-.01x + 900 = 800 (combining like terms)

-.01x = -100 (subtracting 800 from both sides of equation)

x = $10,000 (dividing both sides by .01)

Step 5:

x = $10,000 = amount invested at 2%

$30,000 – x = $30,000 - $10,000 = $20,000 = amount invested at 3%