Calc 1

1. More Americans are buying organic fruit and vegetables and products made with organic ingredients. The amount, A, in billions of dollars, spent on organic food and vegetables t years after 1995 is approximated by $A = 2.43e^{0.18t}$.

   a. Use the model to estimate the amount Americans spent on organic food and beverages in 2009. 30.2 billion

   b. Use the model to estimate the year in which Americans spent $3 billion on organic food and beverages. 1997

   c. Find dA/dt. $0.4374e^{0.18t}$

2. Students in a zoology class took a final exam. They took equivalent forms of the exam at monthly intervals thereafter. After t months, the average score, S, as a percentage, was found to be approximated by $S = 78 - 15 \ln (t + 1)$ where $t \geq 0$.

   a. What was the average score when they initially took the test? 78%

   b. What was the average score after four months? 53.86%

   c. How many months went by before the average test score dropped to 60%? 2.32 ≈ 3 months

   d. Find dS/dt. dS/dt = -15/(t+1)

3. In 2007, United States per capita income, I, was $46,459. In 2011, it had grown to $47,153. Assume that the growth in United States per capita income follows an exponential model.

   a. Write the function which models this behavior; let t = 0 apply to 2007. $I = 46459e^{0.0037t}$

   b. Use the model to predict the United States per capita income in 2020. $48,748.29

   c. In what year will United States per capita income double that of 2007? 2195

4. Atmospheric pressure P, in pounds per square inch, at an altitude a, in feet, is approximated by $P = P_0e^{-0.00005a}$, where P_0 is the pressure at sea level. Assume $P_0 = 14.7$.

   a. Find the atmospheric pressure at an altitude of 10,000 feet. 8.916 lbf/in^2

   b. At what altitude is the atmospheric pressure approximately 7.35 pounds per square foot? 13862.94 ft

c. Find dP/da. $dP/da = -0.000735e^{-0.00005a}$

5. The number of non-farm proprietorships, N, in thousands, in the United States, can be modelled by N = 8,400 ln t – 10,500 where t is the number of years since 1970.

a. Use the model to predict the number of non-farm proprietorships in the United States in 2014. 21288

b. Use the model to predict the year in which the number of non-farm proprietorships surpassed 10,000. 1982

c. Find dN/dt. dN/dt = 8400/t

11. Consider $y = 2 + 3x^2 – x^3$.

a. State the domain. {x|x ϵ ℝ} x(-∞, ∞)

b. Find the y-intercept. (0,2)

c. Find the x-intercept(s). (3.196,0)

d. Is the function even? No

e. Is the function odd? No

f. State the equation of the horizontal asymptote, if any. None

g. State the equation of the slant asymptote, if any. None

h. State the equation of the vertical asymptote, if any. None

i. State the interval(s) on which the function is decreasing. x=-∞ to x=0, x=2 to x=∞

j. State the interval(s) on which the function is increasing. x=0 to x=2

k. Find the local maximum value(s). y=6

l. Find the local minimum value(s). y=2

m. Find the inflection point(s). (1,4)

n. Find the interval(s) on which the function is concave down. x=1 to x=∞

o. Find the interval(s) on which the function is concave up. x=-∞ to x=1

p. Sketch the graph of the curve on the axes below and to the left.

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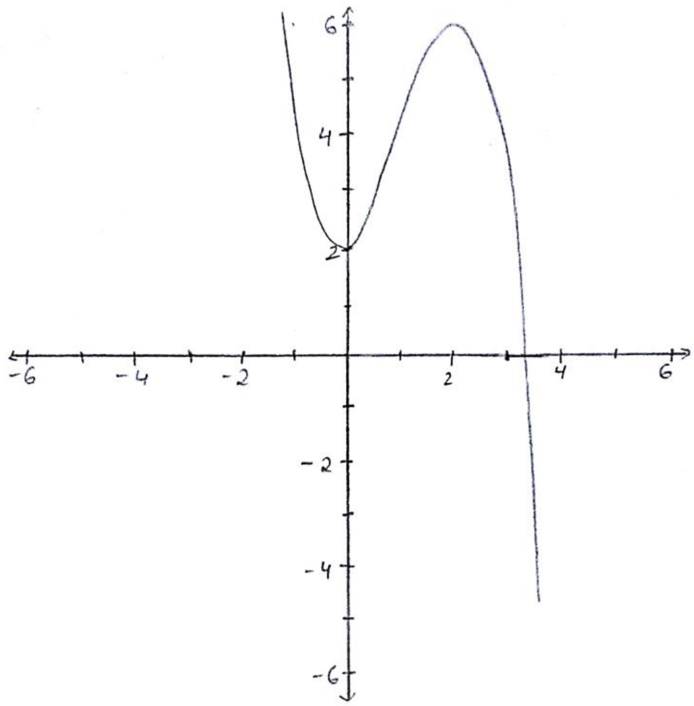

12. Consider $y = \dfrac{1}{x^2 - 4}$.

a. State the domain. $\{x \neq 2, -2\}$

b. Find the y-intercept. (0, -0.25)

c. Find the x-intercept(s). None

d. Is the function even? Yes

e. Is the function odd? No

f. State the equation of the horizontal asymptote, if any. y=0

g. State the equation of the slant asymptote, if any. None

h. State the equation of the vertical asymptote, if any. x=2 and x=-2

i. State the interval(s) on which the function is decreasing. (0,2) and (2,∞)

j. State the interval(s) on which the function is increasing. (-∞,-2) and (-2,0)

k. Find the local maximum value(s). y=-0.25

l. Find the local minimum value(s). None

m. Find the inflection point(s). None

n. Find the interval(s) on which the function is concave down. (-2,2)

o. Find the interval(s) on which the function is concave up. (-∞,-2)

p. Sketch the graph of the curve on the axes below and to the left.

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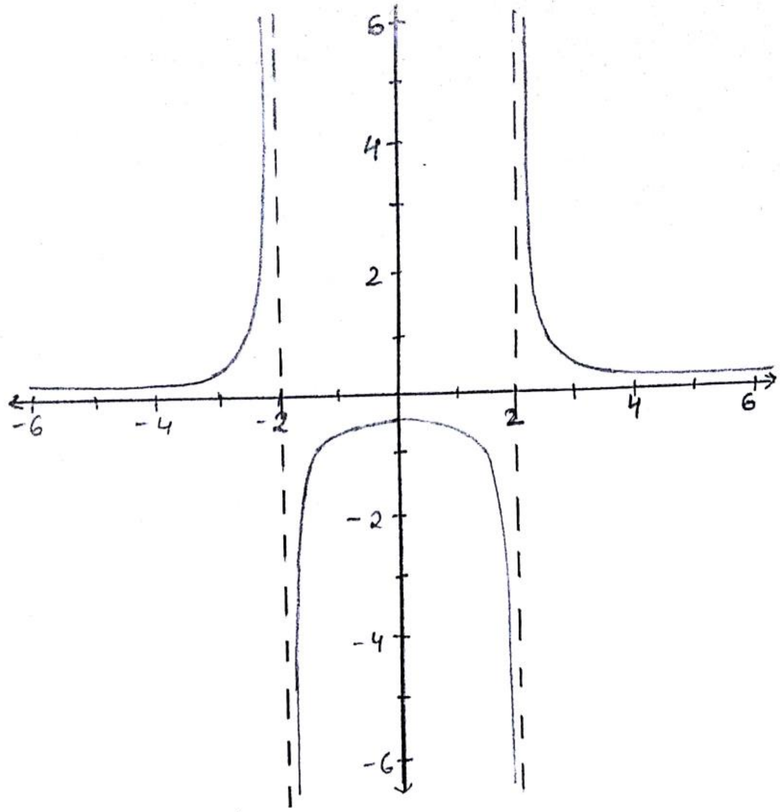

13. Consider $y = (1 - x)e^x$.
   a. State the domain. <u>{x|x ϵ ℝ} x(-∞, ∞)</u>
   b. Find the y-intercept. <u>(0,1)</u>
   c. Find the x-intercept(s). <u>(1,0)</u>
   d. Is the function even? <u>No</u>
   e. Is the function odd? <u>No</u>
   f. State the equation of the horizontal asymptote, if any. <u>None</u>
   g. State the equation of the slant asymptote, if any. <u>None</u>
   h. State the equation of the vertical asymptote, if any. <u>None</u>
   i. State the interval(s) on which the function is decreasing. <u>(0, ∞)</u>
   j. State the interval(s) on which the function is increasing. <u>(-∞, 0)</u>
   k. Find the local maximum value(s). <u>y=1</u>
   l. Find the local minimum value(s). <u>None</u>
   m. Find the inflection point(s). <u>(-1, 2/e)</u>
   n. Find the interval(s) on which the function is concave down. <u>(-1, ∞)</u>
   o. Find the interval(s) on which the function is concave up. <u>(-∞, -1)</u>
   p. Sketch the graph of the curve on the axes below and to the left.

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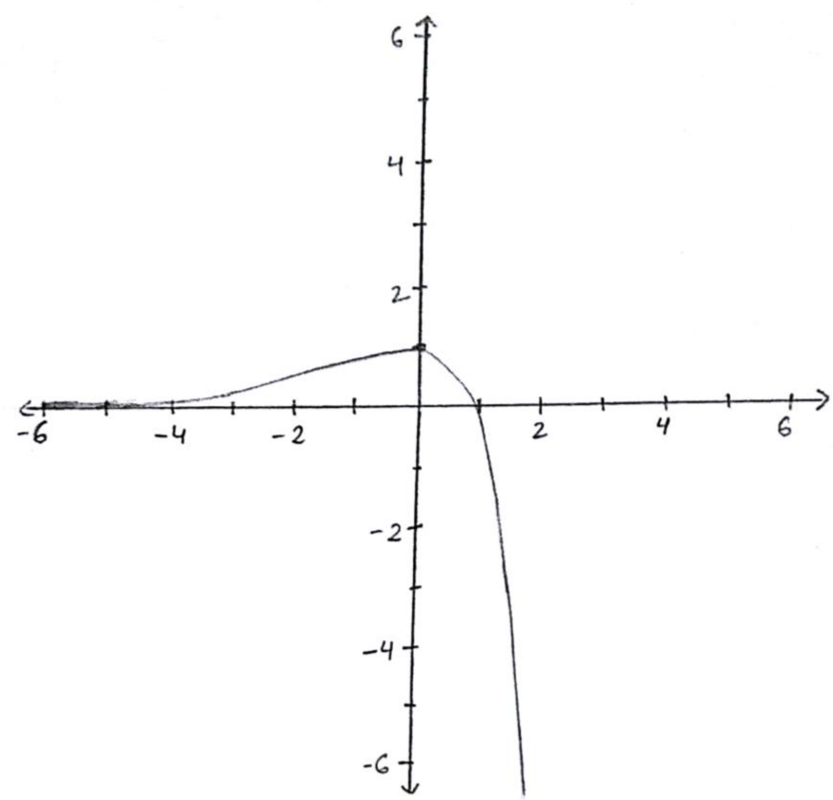

14. Inge is flying her kite in a wind that is blowing it east at a rate of 25 feet per second. The kite is flying 300 feet above her hand.

a. What is the equation which relates the horizontal distance, x, and the vertical distance, y, to the length of the string, r, between Inge and her kite? $r^2 = x^2 + y^2$

b. Find dr/dt. $dr/dt = 625t/\sqrt{(300^2 + 625t^2)}$

c. When the length of the string is 500 feet, find the horizontal distance between Inge and her kite. 400 feet

d. At the instant the length of the string is 500 feet, how fast must Inge let out string to keep the kite flying with the same speed and altitude? 20 feet per second

15. A ladder 10 feet long rests against a vertical wall.

a. What is the equation which relates the horizontal distance, x (from the bottom of the wall to the bottom of the ladder) to the vertical distance, y (from the bottom of the wall to the top of the ladder)? $x^2 + y^2 = 100$

b. How high above the ground is the top of the ladder when the bottom of the ladder is six feet from the base of the wall? 8 feet

c. Find dy/dx. $dy/dx = -x/y$

d. If the bottom of the ladder slides away from the wall at a rate of one foot per second, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is six feet from the wall? 0.75 feet per second